# Tasks 9&10. The Thracian Tomb

After the discovery of a large number of Thracian tombs in the "Valley of Roses" in central Bulgaria, the archeologists started calling this area the "Valley of the Thracian Kings". Many of the tombs are situated near Seuthopolis, the capital of the Odrysian kingdom since 320 BC. The cemetery of Seuthopolis included a number of tholos tombs, in which the upper-class were interred, sometimes along with their horses*. These vaulted "beehive" tombs comprise a narrow corridor and a round burial chamber covered by tumuli.

A plenty of small artificial hills are spread all over the valley and it is believed that most of them hide Thracian tombs. Treasure hunters are constantly undertaking small scale excavation works plundering the mounds. However, the Bulgarian Ministry of Culture cannot afford ubiquitous excavation works to prevent the plundering. They need to know with certainty whether a hill is a tumulus or not, to save time and money and forestall the treasure hunters.

You have been invited by the Bulgarian Ministry of Culture to help them in determining the archeological value of the hills. You've accepted. Your idea is to use a small EM wave generator and a receiver (a hand-made GPR) to probe the subterranean recesses. The EM generator emits a very narrow beam, with a cross-section smaller than the receiving aperture of the receiver.

**Briefly explain the principle of operation of the Ground Penetrating Radar (GPR):**

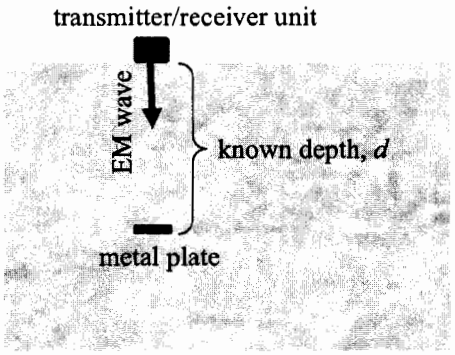
• In order to use the GPR, one drags the sender and reciever apparatus along the ground, and based upon the signal strength of the returning wave, one can detect the presence and depth of an object.

* In book 5, Herodotus describes the customs of various Thracian tribes.

> *The Thracians who live above the Crestonaeans observe the following customs. Each man among them has several wives; and no sooner does a man die than a sharp contest ensues among the wives upon the question which of them all the husband loved most tenderly; the friends of each eagerly plead on her behalf, and she to whom the honor is adjudged, after receiving the praises both of men and women, is slain over the grave by the hand of her next of kin, and then buried with her husband. The others are sorely grieved, for nothing is considered such a disgrace.*
> *Their wealthy ones are buried in the following fashion. The body is laid out for three days; and during this time they kill victims of all kinds, and feast upon them, after first bewailing the departed. Then they either burn the body or else bury it in the ground. Lastly, they raise a mound over the grave, and hold games of all sorts, wherein the single combat is awarded the highest prize. Such is the mode of burial among the Thracians.*

Before you start the real work, you need to undertake some preliminary measurements that will allow you to determine the soil parameters. You know that the conductivity, $\sigma$, and the permittivity, $\varepsilon$, strongly depend on the composition of the soil and the recent weather conditions, so you cannot use averaged values reported in books and technical papers.

You decide to dig a pit and bury a metal plate at certain depth, $d$, and then place the transmitting/receiving unit directly above the plate on the earth surface, shown in the figure to the right. By measuring the received signal, $E_R$, and the time, $\tau$, it needs to reach the receiver, you hope to be able to deduce the soil parameters, eventually.




**Briefly explain what happens to the emitted by the transmitter EM wave and provide the math expression that relates the received signal strength to the depth, at which the metal plate is.**

The emitted wave travels downward towards the metal plate, reflects off it, and returns to the transmitter. Along the way the wave is attenuated due to soil being mildly conductive. Therefore the mathematical expression for the recieved signal is: $E_R = E_T e^{-\alpha 2d}$.

# Task 9. The Thracian Tomb: Preliminary Measurements

## Questionnaire

Give brief but accurate and thorough explanation if necessary. Provide math expressions where needed to show how you've obtained the particular result.

1. What is the depth at which you buried the metal plate? What are the values of the received signal strength and the signal delay?

The depth is 1.5m
The received signal strength is 172 mV/m
The signal delay is 30 ns.

2. What is the attenuation constant for the wave in the soil? Provide the math expression and the value.

$$\alpha = \omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right]^{1/2} = \omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]\right]^{1/2}$$

or

$$E_R = E_T e^{-\alpha 2d} \Rightarrow 172 = 200\, e^{-\alpha[2\times 1.5\text{m}]} \Rightarrow \boxed{\alpha = 0.503}$$

3. What is the speed of the wave in the soil? Provide the math expression and the value.

$$u_p = \omega/\beta = \left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right]^{1/2} = \left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]\right]^{1/2}$$

or

$$T = 2d/u_p \Rightarrow 30\text{ns} = \frac{2[1.5\text{m}]}{u_p} \Rightarrow \boxed{u_p = 1\times 10^8 [\text{m/s}]}$$

4. What is the phase constant of the wave in the soil? Provide the math expression and the value.

$$\beta = \omega\left[\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right]^{1/2} = \omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]\right]^{1/2}$$

or

$$\beta = \frac{\omega}{u_p} \Rightarrow \beta = \frac{2\pi(300\text{MHz})}{1\times 10^8\,\text{m/s}} = \boxed{6\pi \approx 18.8496}$$

5. What is the loss tangent of the soil at the frequency used in the measurements? Provide the math expression obtained with MATLAB and the value.

$$\frac{\alpha^2}{\beta^2} = \frac{\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1}{\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1} = \frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1}{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1}$$

$$\boxed{\chi = \frac{-2\alpha\beta}{\alpha^2-\beta^2}}$$

6. What type of medium is the soil at the measurements site at 300 MHz, lossy or lossless, and why? In case it is lossy, specify whether the soil is a low-loss dielectric, a good conductor, or neither.

The soil is a lossy medium because $\alpha$ is greater than zero.

$\chi = \frac{\varepsilon''}{\varepsilon'} = 0.0053 < 0.01$ therefore the soil is a low loss medium.

7. What is the relative permittivity of the soil? Provide the math expression and the value.

$\beta = \omega\sqrt{\mu\varepsilon} \Rightarrow 6\pi = 2\pi(300\times10^6)\sqrt{4\pi\times10^{-7}\,\varepsilon_r\,(8.854\times10^{-12})} \Rightarrow$

$$\boxed{\varepsilon_r = 8.987}$$

8. What is the conductivity of the soil? Provide the math expression and the value.

$\frac{\varepsilon''}{\varepsilon'} = 0.0053 = \frac{\sigma}{\omega\varepsilon} \Rightarrow 0.0053 = \frac{\sigma}{2\pi(300\times10^6)(8.987\times8.854\times10^{-12})} \Rightarrow$

$$\boxed{\sigma = 7.95\times10^{-4}\ [S/m]}$$

9. What is the maximum depth, at which the metal plate can be placed provided that such a deep pit could be dug?

$E_R = E_T e^{-\alpha 2d}$

$20\,mV/m = (200\,mV/m)\, e^{-0.503(2d)}$

$\ln\left(\frac{1}{10}\right) = -1.006\,d$

$$\boxed{d = 2.289\ m}$$

6. Don't forget that the received signal is in dB. Denote the received signal with $S$ and give $S$ as a function of the object depth.

$E_R = 0.5 E_T e^{-\alpha 2d}$

$$S = 20\log_{10}(0.5e^{-\alpha 2d})$$

7. Solve the equation from the previous question with respect to the depth (distance from the earth surface).

$10^{(S/20)} = 0.5e^{-\alpha 2d}$

$-\alpha 2d = \ln(10^{(S/20)}/0.5)$

$$d = -\frac{1}{2\alpha}\ln(10^{(S/20)}/0.5)\ [m]$$

8. Which of the hills hide large archeological objects in their recesses?

Hill 2 Hill 5
Hill 3 Hill 6

9. What is the maximum depth at which an air cavity can be detected using your EM-wave generator? Provide the math expression and the value. Compare this value with the value from question 9, Task 9, and comment.

$20\,mV/m = 0.5(200\,mV/m)e^{-0.503(2d)}$

$$\frac{-\ln(0.2)}{2(0.503)} = d$$

$d = 1.5998\ [m]$ this is less than the distance in Task 9. This makes sense because only half of the wave is reflected off the air/soil boundary, while approximatly all of the wave was reflected off the metal plate.